STANDARD
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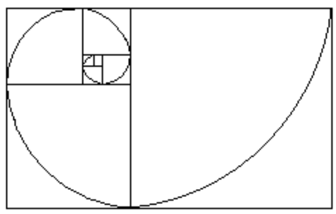
Setting the Standard

"Fibonacci"

The man & the markets



STANDARD & POOR'S

ECONOMIC RESEARCH PAPER

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Contents

Fibonacci – The man & the markets

Everybody with exposure to the commentary of technical analysts has come across the name Fibonacci or its common abbreviation, Fibo. We hear of Fibonacci retracements, Fibonacci levels, Fibonacci targets and Fibonacci fans among others. These Fibonacci comments are often supplied with a number such as 0.382 or 0.618. We are told that Fibonacci can help to forecast support and/or resistance as well as to suggest potential target levels for existing trends. Some analysts even use Fibonacci in efforts to calculate the timing of future market turning points.

So who is Fibonacci and how does he help with all these tasks?

FIBONACCI – THE MAN

<u>The many names of Leonardo</u>

No one knows the exact date of Fibonacci's birth with certainty but it is believed to have been in the eighth decade of the twelfth century[i], although most sources settle for a date of circa 1170. Similarly, the date of his death is not known with any certainty although the lack of any record of his life after 1240 suggests he died sometime in the decade thereafter.

Not only have the precise dates of his birth and death been lost over the centuries but even the name (or names) by which he called himself and by which he was known to his contemporaries, are now the subject of conjecture. The only point that everybody appears to agree on is that his given name was Leonardo and he was born in the city-state of Pisa, thus he was known as Leonardo Pisano. Just when the name Fibonacci appeared is uncertain.

Leonardo's father was named Guilielmo (or Guglielmo depending on the source). He was a descendant of Bonacci (or Bonaccio) who was probably Leonardo's grandfather. It appears that both Leonardo and his father identified themselves as descendants of Bonacci with Leonardo's father known as Guilielmo Bonacci (or Guglielmo Bonaccio). In Latin, the term "filius Bonacci" means "son of Bonacci" and some believe that Fibonacci is simply an abbreviated form of "filius Bonacci". Ironically, "Bonacci" is believed to have meant "simpleton".[ii]

Guillaume Libri wrote in "Histoire des sciences mathematiques en Italie" (1838)[iii];

> "*Fibonacci is a shortened form of filius Bonacci, contractions of which are widely exemplified in the making of the surnames of Tuscan families. There is no hint of evidence that Leonardo of Pisa referred to himself as Fibonacci nor that he was ever so named by his contemporaries.*"

Some suggest that Libri was in fact the originator of the name Fibonacci while others such as Victor J. Katz[iv] suggest the name was given to Leonardo by Baldassarerre Boncompagni, "the nineteenth century editor of his (Leonardo's) works". This opinion is given a slight twist on an internet web site which is part of the University of Surrey[v] where it is claimed that, "… two early writers on Fibonacci (Boncompagni and Milanesi) regard Bonacci as the family name so that Fib-Bonacci is like the English names of Robin-son or John-son". In the sentence prior to this, the web site proclaims in no uncertain terms, "He called himself Fibonacci…" This is supported by two other sources including The Encyclopaedia Britannica. On brittanica.com it is stated that the original name of Leonardo Pisano was Leonardo Fibonacci.[vi] The other source mentioned is the author of "Fascinating Fibonaccis", Trudi Hammel Garland, who gives the story one more twist. She claims that, "he nicknamed himself 'Fibonacci' for writing purposes", citing the example of Mark Twain (whose real name was Samuel Clemens).

Further evidence in the search for the origin of the name Fibonacci comes from D. E. Smith,[vii] who says,

> "*when Leonardo himself wrote 'filius Bonacci', 'filius Bonaccij' and 'filius Bonacii', he knew what the words would mean to Latin readers.*"

This seems the most logical solution. Although Leonardo may not have actually used the name Fibonacci as we do today, he certainly meant to convey the impression that he was a descendant (son) of Bonacci. And, according to Libri's comments about Tuscan family names, Leonardo would not have been surprised to see 'filius Bonacci' abbreviated to Fibonacci.

But wait, there's more. In one of his books, Flos, believed to date from 1225, Leonardo referred to himself as "Leonardo Bigollo Pisano" and a decree made by the Republic of Pisa in 1240 refers to "Master Leonardo Bigollo". Here too, there are conflicting opinions with interpretations regarding the meaning of 'Bigollo' ranging from 'good-for-nothing' to 'traveller'.[viii] There is no doubt however, that Leonardo did use this name.

The early years

Leonardo's life (and arguably our own) was significantly influenced by his father's career.
In his most famous book, Liber Abaci (1202, revised in 1228), Leonardo tells us;

> *"When my father, who had been appointed by his country as public notary in the customs at Bugia acting for the Pisan merchants going there, was in charge, he summoned me to him while I was still a child, and having an eye to usefulness and future convenience, desired me to stay there and receive instruction in the school of accounting."*[ix]

Bugia was later known as Bougie and is now known as Bejaia, a coastal town in northeastern Algeria. While living here and travelling with his father on business, Leonardo's interest in mathematics flourished and he expanded his knowledge well beyond the Roman Numerals that remained dominant in Europe at that time.

With exposure to traders, merchants and tutors from all over the Arab world, northern Africa, the Mediterranean and even India, Leonardo learned of many different mathematical systems. He soon realized that the Hindu-Arabic decimal positional system had many advantages over Roman Numerals. Previously complex calculation methods for addition, subtraction, multiplication and division were greatly simplified by the use of a positioning zero and the assignment of different values to the individual symbols (numbers) depending on their relationship to one another, e.g. the fact that 155 has very different value to both 551 and 50501.

Many of Leonardo's ideas in relation to the Hindu-Arabic decimal positional system had been laid out several centuries earlier by the Persian mathematician, astronomer and author known as Abu Ja'far Mohammed bin Musa al-Khowarizmi in his book "Kitab al jabr w'al-muqabala", written in approximately 825 A.D. It is from the title of this book that we derive our word **algebra** ("al jabr") and from the name of the author that we derive the word **algorithm**, which was originally algorism.

Return to Pisa: Leonardo, the author

The next phase of Leonardo's life began with his return to Pisa sometime around the turn of the century. Shortly afterwards (1202) he wrote his most famous book, *Liber Abaci*, which means Book of the Abacus or Book of Calculating. This book set out the methods and rules of performing various calculations with the Hindu-Arabic decimal positional system using Arabic numerals. Some modern day authors suggest that with this book Leonardo single-handedly introduced Europe to this superior system. While this is probably not entirely correct, as there is some evidence of al-Khowarizmi's writings having been translated for Europeans, there is no doubt that Leonardo's work was a significant influence in the move away from Roman Numerals.

In addition to explaining the principles of this new system, *Liber Abaci* included numerous practical examples relating to currency exchange, calculating profit, weights, measures and other similar issues that were of interest to the merchants of the day. Although merchants in some areas initially had an aversion to the use of Hindu-Arabic numerals and banned them through fear of being exploited, the new system eventually gained acceptance and spread slowly but surely with the merchants throughout Europe. *Liber Abaci* also contained considerable detail on "speculative mathematics, proportion, the Rule of False Position, extraction of roots, and the properties of numbers, concluding with some geometry and algebra."[x]

Liber Abaci was widely copied (by hand) but it was by no means Leonardo's only contribution to the world of mathematics. Other works by Leonardo still in existence include *Practica Geometriae* (1220/21), *Flos* (1225) and *Liber Quadratorum* (1225). Unfortunately however, some of his writings including "his book on commercial arithmetic *Di Minor Guisa* (are) lost as is his commentary on Book X of Euclid's *Elements* which contained a numerical treatment of irrational numbers which Euclid had approached from a geometric point of view."[xi]

The later years

The practical applications of Leonardo's work saw him attract considerable fame among his contemporaries and the scholars of the day. In particular he came to the attention of Frederick II, the Holy Roman Emperor (1220-1250), King of Germany (1212-1250) and King of Sicily (1197-1250). During a court visit to Pisa, believed to have taken place in 1225, Leonardo was summoned to meet with Frederick II and a number of his scholars. During this meeting (believed to be a type of mathematics tournament) Leonardo was presented with a number of mathematical problems by Johannes of Palermo. An example of one of the questions posed to Leonardo is given in Trudi Hammel Garland's book, *Fascinating Fibonaccis*, as follows;

> *"Find a rational number (whole number or common fraction) such that when 5 is added to its square or subtracted from its square, the result (in either case) is the square of another rational number… Without the benefit of a calculator or computer, Fibonacci arrived at the correct answer: 41/12."*

Leonardo later included several of the answers to the questions posed by Johannes of Palermo in his 1225 book *Flos*. Another of these problems included finding "an accurate approximation to a root of $10x + 2x^2 + x^3 = 20$."[xii] According to the University of St Andrews web-site, Leonardo proved "that the root of the equation is neither an integer nor a fraction, nor the square root of a fraction…. Without explaining his methods, Fibonacci then (gave) the approximate solution in sexagesimal notation as 1.22.7.42.33.4.40 (this is written to base 60, so it is $1 + 22/60 + 7/60^2 + 42/60^3 + \ldots$). This converts to the decimal 1.3688081075 which is correct to nine decimal places, a remarkable achievement."

These very brief examples give a glimpse of Leonardo's remarkable mathematical ability. Despite his brilliance however, Leonardo is today best remembered today not for his largely overlooked contributions to number theory but instead for the "rabbit problem" which appeared as one of a number of problems in the third section of Liber Abaci.

THE FIBONACCI SEQUENCE – OR – THE RABBIT PROBLEM

One of the examples used by Leonardo to demonstrate the application of Hindu-Arabic numerals in *Liber Abaci* stands out clearly from the others and gives rise to what we know today as the Fibonacci Sequence. Although the exact wording varies from one source to another, both the University of St Andrews web-site and britannica.com use the following version;

> *"A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month becomes productive?"*

At the start of the first month we will have only the original pair. By the start of the second month the original pair will have multiplied and there will be a total of two pairs of rabbits. During the second month the original pair will multiply again while the second pair matures, thus at the start of the third month there will be three pairs. Of these, two pairs will multiply during the third month with the result that we will have five pairs by the start of the fourth month. This is best shown in a tabular format.

Start of Month	Adult Pairs	New Pairs	Total number of Pairs
1	1	-	1
2	1	1	2
3	2	1	3
4	3	2	5
5	5	3	8
6	8	5	13
7	13	8	21
8	21	13	34
9	34	21	55
10	55	34	89
11	89	55	144
12	144	89	233
End of month 12	233	144	377

In answering this question, it appears that Leonardo considered the original pair of rabbits to be adults that were able to multiply during their first month in the enclosure. This would explain why he omitted the first number "1" from the sequence as we know it today, which begins with 1, 1, 2, 3, 5, 8… and continues expanding infinitely.

Position in Sequence	1	2	3	4	5	6	7	8	9	10	11	12
Fibonacci Number	1	1	2	3	5	8	13	21	34	55	89	144

This recursive number sequence can be expressed by the following formula;

$X_{n+1} = X_n + X_{n-1}$ which can be rearranged to give; $X_n = X_{n-1} + X_{n-2}$

where X_n = number of pairs of rabbits after n months.

Starting with $n = 1$ it is clear that $n - 1 = 0$ and so 0 is also sometimes shown as a part of the sequence.

It was the French born mathematician, Edouard Lucas (1842 – 1891) who coined the term "Fibonacci Sequence". There is only a minor difference between this sequence and Lucas Numbers, with the latter being a very similar sequence that starts with the numbers 1 and 3 and continues with 4 (=1+3), 7 (=3+4), 11 (=4+7), 18 (=7+11) and so on.

THE FIBONACCI RATIO(S) – OR – THE GOLDEN RATIO/MEAN/SECTION

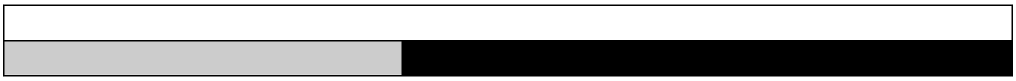
Before looking at how the particular numbers of the Fibonacci sequence apply to analysis of the markets we need to look beyond the individual numbers and consider the relationships between these numbers.

As the Fibonacci sequence progresses a clear pattern becomes apparent in the relationship between numbers in the sequence and the next number up or down in the sequence. The ratio of any number in the sequence (after the first half dozen or so) to the next higher number is approximately 0.618 while the ratio to the next lower number is approximately 1.618 with accuracy increasing as the sequence progresses. Mathematicians would recognize these ratios as *phi* (0.618) and *Phi* (1.618). A more accurate method of calculation of these ratios is to take the square root of 5 then either add 1 (for *Phi*) or subtract 1 (for *phi*) and divide the result by 2.

Another commonly referred to ratio is that between any number in the sequence and the number that is two higher in the sequence, which tends toward approximately 0.382 as the sequence progresses.

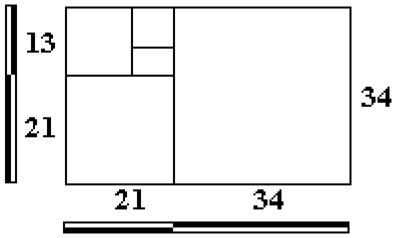
This ratio of 1.618 is also known as the Golden Ratio. It is worth noting that any two randomly selected numbers can be used to begin a recursive sequence (using $X_n = X_{n-1} + X_{n-2}$) that will generate *Phi* & *phi*. This suggests that it is not the Fibonacci numbers themselves that are important to us, but rather the relationship between the Fibonacci numbers. However, because the Fibonacci sequence starts with 1 and 1, "the starting point of mathematical growth"[xiii], it can be argued that they do indeed contain special characteristics. This will be explained further when we look at the relationship between Fibonacci and the Elliott Wave principle.

These ratios (*phi & Phi*) appear again in The Golden Section. The basic concept here is that any length can be divided in such a way that the ratio between the smaller part and the larger part is the same as the ratio between the larger part and the whole. Mathematically; S/L = L / (S + L)



Small part = 0.618 of large part Large part = 0.618 of the whole

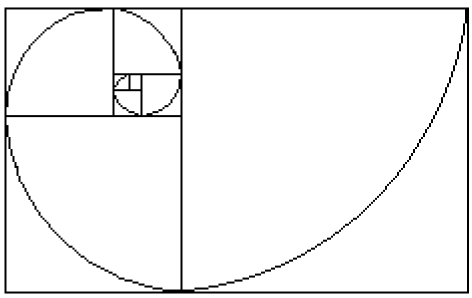
The Golden Ratio/Section can be used to construct a Golden Rectangle as shown below.



The ancient Egyptians were apparently aware of the existence of the Golden Ratio and even incorporated it in the construction of the Great Pyramid at Gizeh. The length of each sloping side (or apothem) of this pyramid is equal to 1.618 times half its base. The ancient Greeks also applied this ratio in their construction of the Parthenon. Even today, modern architects make use of the Golden Ratio with Le Corbusier's contributions to the United Nations building in New York being one famous example. Many artists are also influenced by the Golden Ratio with great masters including both Giotto and da Vinchi incorporating the ratio in their work. It is not only artists that are aware of the aesthetically pleasing qualities of the Golden Ratio and the Golden Rectangle. In approximately 1871 the psychologist Gustav Fechner asked some 347 subjects which of ten different quadrangles they found most pleasing. The quadrangle that was deemed as most pleasing by the highest number of subjects was the quadrangle that most clearly resembled a Golden Rectangle. It is no coincidence that we now see many quadrangles approximating the Golden Rectangle in everything from business cards to movie theatre screens and billboards. Even the dimensions of your kitchen toaster and microwave oven may approximate the Golden Rectangle!

The Golden Section/Ratio appears throughout nature and has been described as one of the building blocks of natural growth patterns.

As mentioned above, the Golden Ratio can be used to construct a Golden Rectangle. By drawing an arc between the corners of each of the infinitely shrinking/expanding squares within the rectangle we can create an equiangular or logarithmic spiral, also known as the Golden Spiral. Nature abounds with such spirals but perhaps the most obvious example is the chambered Nautilus shell.



The constant shape of this spiral both at microscopic and inter-galactic levels has similarities with "chaos theory" and the Mandlebrot Set, which while differing from the spiral, also maintains a constant shape when magnified.

As Frost & Prechter stated in their 1978 book, Elliott Wave Principle, "… a spiral implies motion: growth & decay, expansion and contraction, progress and regress. The logarithmic spiral is the quintessential expression of natural phenomena found throughout the universe." They go on to give other examples of natural occurrences of the Golden Spiral including the tail of a comet spiraling away from the sun, the web of the epeira spider, the growth patterns of bacteria, the microscopic formation of quasi crystals, pinecones, seahorses, animal horns, the arrangement of seeds on a sunflower, hurricane clouds, whirlpools and galaxies.

Even the human body can be shown to contain elements of the Golden Ratio which is the basic building block of the Golden Spiral. On average, the distance from the navel to the top of the head is 0.618 of the distance from the feet to the navel while the distance from the feet to the navel is on average 0.618 of the total height of a person. On a smaller scale, the 5 digits of each hand are each constructed with 3 bones (Fibonacci numbers). These three bones of each digit (finger) can also be shown to display the properties of the Golden Section. On a microscopic scale, the basic building blocks of human beings, the DNA double helix, also contains the Golden Ratio.

Frost & Prechter make the case for relating the Golden Ratio (and hence the Fibonacci Sequence) to the financial markets with the following observation;

> *"If phi is the growth-force in the universe, might it be the impulse behind the progress in man's productive capacity? If phi is a symbol of the creative function, might it govern the creative activity of man? If man's progress is based upon production and reproduction "in an endless series," is it not possible, even reasonable, that such progress has the spiraling form of phi, and that this form is discernible in the movement of the valuation of his productive capacity, i.e., the stock market?"*

APPLICATION OF FIBONACCI NUMBERS & THE GOLDEN RATIO IN THE FINANCIAL MARKETS

How then do we draw a connection between Fibonacci and the constant ebb and flow of the markets? There are two areas to consider. The first involves the Fibonacci numbers themselves. The second is based on the application of the Golden Ratio.

<u>Fibonacci numbers in financial markets</u>

As briefly mentioned earlier, there is a connection between Fibonacci and Elliott Wave Theory. Ralph Nelson Elliott (1871–1948) (pictured) pursued his keen interest in stock market movements after moving to California to recuperate from the anemia which had plagued him during his time as a corporate accountant in Guatemala. With time on his hands Elliott undertook an in depth analysis of the Dow Jones averages and eventually came up with his "Wave Theory" which he suggested in a letter to Charles Collins would be "a much needed complement to Dow theory".

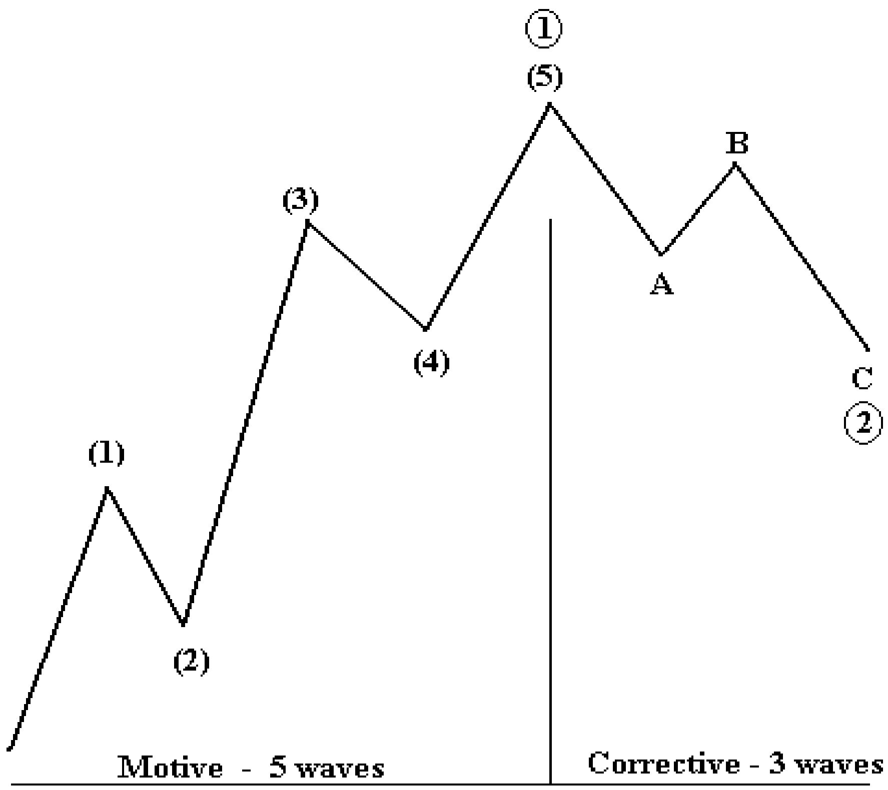
Charles Collins was the publisher of a national market newsletter of the day with an extensive following, which included the admiring R. N. Elliott. Correspondence between the two began in November 1934 when Elliott wrote to Collins suggesting that his "discoveries" might be of interest to Collins and that they could even materially benefit the prestige of Collins' market newsletter. Collins was impressed by the accuracy of Elliott's analysis over the following months and invited him to Detroit to explain the process in detail. Although Elliott's insistence that all market decisions should be based on Wave Theory prevented Collins from directly employing Elliott, he did help him to establish an office on Wall Street. In addition, Collins soon after wrote a booklet entitled "The Wave Principle" (1938) and published it under Elliott's name.

Elliott went on to publicize his theory through various letters and magazines including the Financial World magazine. Then, in 1946 Elliott expanded on "The Wave Principle" with a book called "Nature's Law – The Secret of the Universe". It was in this book that Elliott made the connection between his Wave Theory and Fibonacci. Some have suggested that it was Collins who first alerted Elliott to this connection.

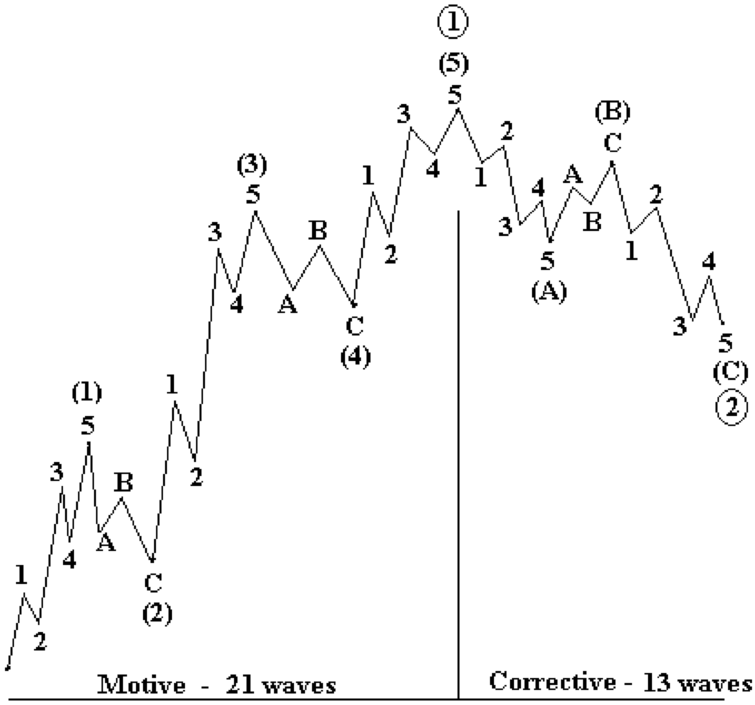
Without delving too deeply into Elliott Wave Theory, Elliott's observations led him to believe that markets move in waves of progress and regress (*motive* and *corrective*). He further stated that periods of progress ultimately take the form of five waves with periods of regress taking the form of three waves. Frost & Prechter explained these movements as follows;

> *"…this is the minimum requirement for, and therefore the most efficient method of, achieving both fluctuation and progress in linear movement. The fewest subdivisions to create fluctuation is three waves. Three waves (of unqualified size) in both directions does not allow progress. To progress in one direction despite periods of regress, movements in the main trend must be at least five waves, simply to cover more ground than the three waves and still contain fluctuation. While there could be more waves than that, the most efficient form of punctuated progress is 5-3, and nature typically follows the most efficient path."* [xiv]

It should be immediately obvious that both 5 and 3 are numbers from the Fibonacci sequence. Wave theory stipulates that each wave can be broken down into sub-waves which will themselves conform to this pattern of 5 waves in the direction of the motive wave of one higher magnitude and 3 waves in the direction of the corrective wave of one higher magnitude. A diagram shows this more clearly.



When each of these waves is broken down into its sub-waves we end up with 21 waves in the main motive wave and 13 waves in the main corrective wave. Again, these are numbers from the Fibonacci sequence.



Numbers from the Fibonacci sequence are also often used as the parameters for calculating various technical indicators. Perhaps one of the most obvious instances of this application of Fibonacci numbers is in the calculation of multiple moving averages. For example, you will often notice the use of 5, 13 & 21 period moving averages, or 21, 34 & 55 period moving averages. In addition to the individual moving averages having a relationship with the underlying price action, the use of Fibonacci numbers will mean that the individual moving averages themselves will have a special relationship to one another through the existence of the underlying Golden Ratio.

The Golden Ratio in financial markets

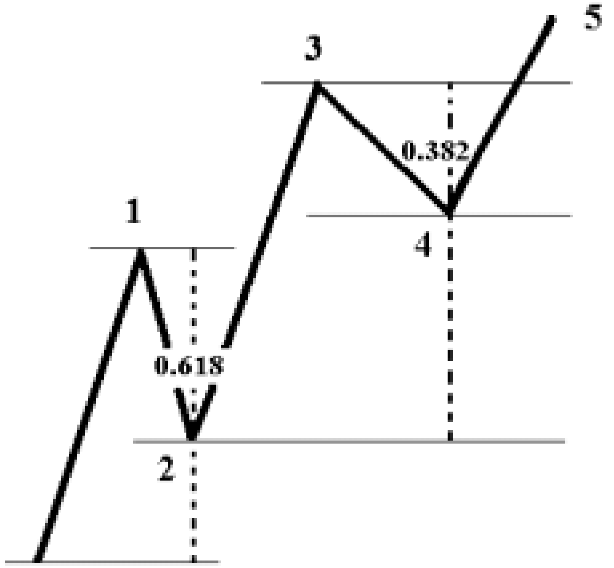
In addition to the appearance in the financial markets of specific numbers from the Fibonacci sequence, observation reveals that many of the "waves" of movement in the market are related to one another either by the Golden Ratio or by some derivative thereof. Some refer to the study of these relationships as Ratio Analysis.

The best known application of the Golden Ratio in the financial markets is what has come to be known as the "Fibonacci Retracement". In simple terms, the Fibonacci Retracement relates to the fact that corrective waves have been observed to retrace the previous wave by either 38.2%, 50% or 61.8% with enough frequency to make this a characteristic worth looking out for.

A pullback of 38.2% is seen as providing a meaningful correction without causing excessive strain on the underlying trend. Pullbacks of this magnitude are often associated with steady, sideways corrections.

A larger pullback of the magnitude of 50% or 61.8% is more often associated with a sharp move against the direction of the prior wave.

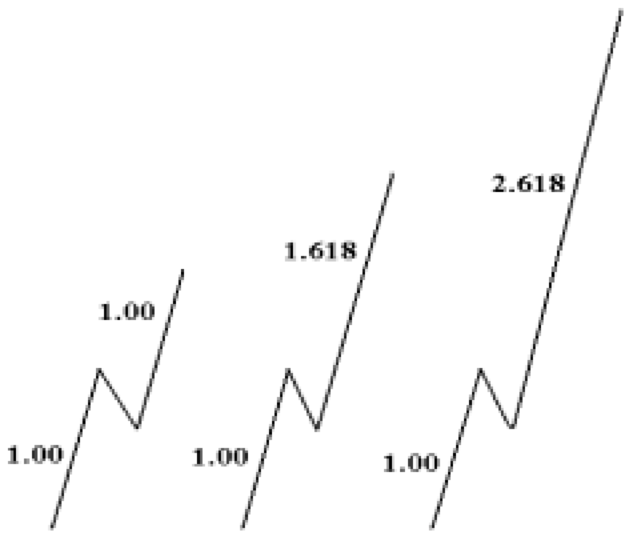
The sketch below shows one example of how such relationships between the waves of the market might appear. In this instance the waves are shown as part of the 5-wave 'motive' wave of Elliott Wave Theory but they are not restricted to being found within this context.



Wave 4 retraces 38.2% of the prior wave, Wave 3. In relative terms this wave can be seen as a sideways correction.

Wave 2 is a sharp correction against the direction of Wave 1 and retraces this wave by approximately 61.8%.

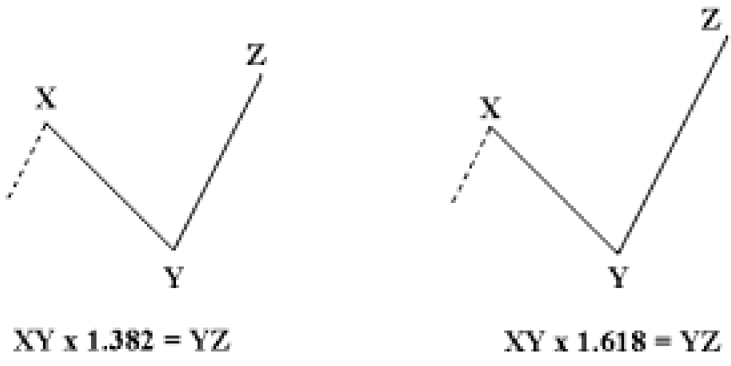
Those willing to look beyond simple retracements will find that the Golden Ratio also regularly appears in the relationship between the lengths of alternate waves moving in the same direction.



Once again there is a connection here with Elliott Wave Theory. Once these simple waves are broken down into waves of a lesser degree it becomes clear that the appearance of these Golden Ratio relationships can be tied in with certain types of wave development.

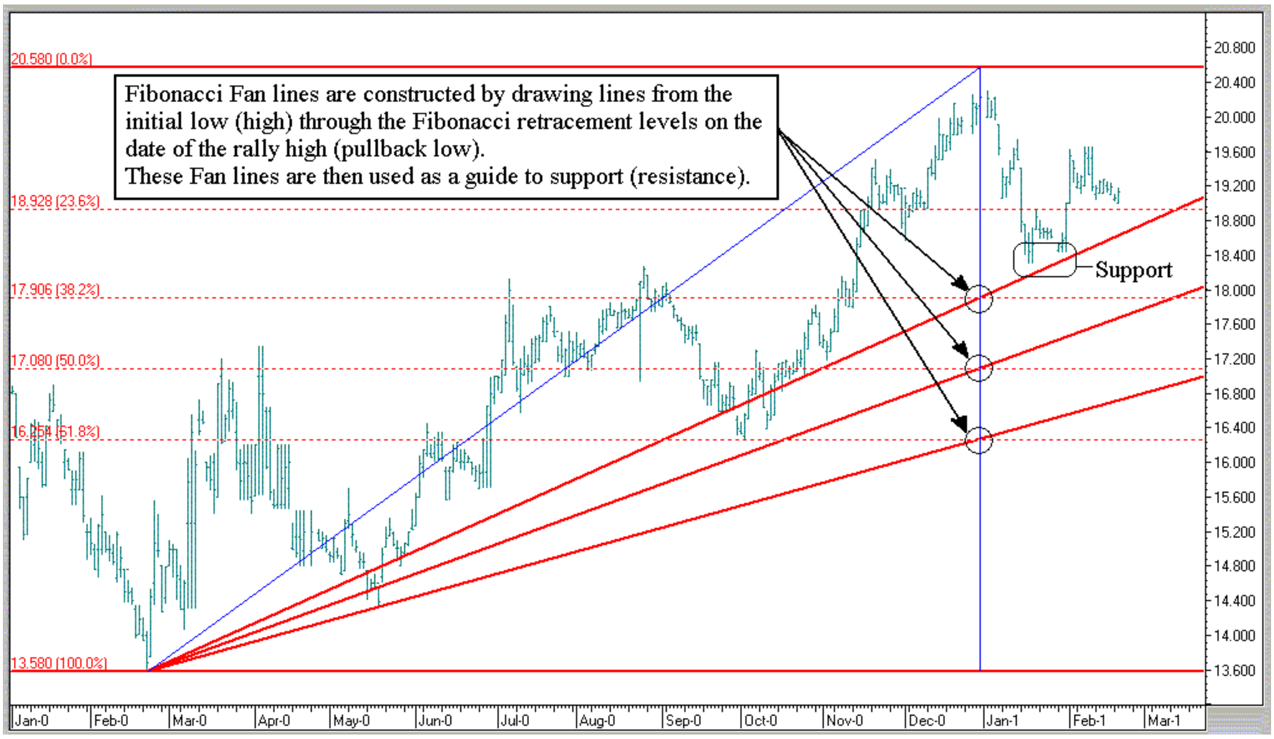
For example, observation has shown that we are most likely to see equal length waves as shown on the left when they are separated by an extended third wave. Again using the 5-wave structure as an example, we are most likely to see a move of a magnitude of either 1.618 or 2.618 as an extended 5th-wave or an extended 1st-wave. However, when dealing with an extended 1st-wave we are more likely to refer to waves 3 to 5 as having a relationship of 0.618 to the 1st-wave.

Another situation in which the Golden Ratio can be found involves a market that retraces in excess of 100% of the previous move thus continuing in the direction of the wave prior to the wave being retraced.



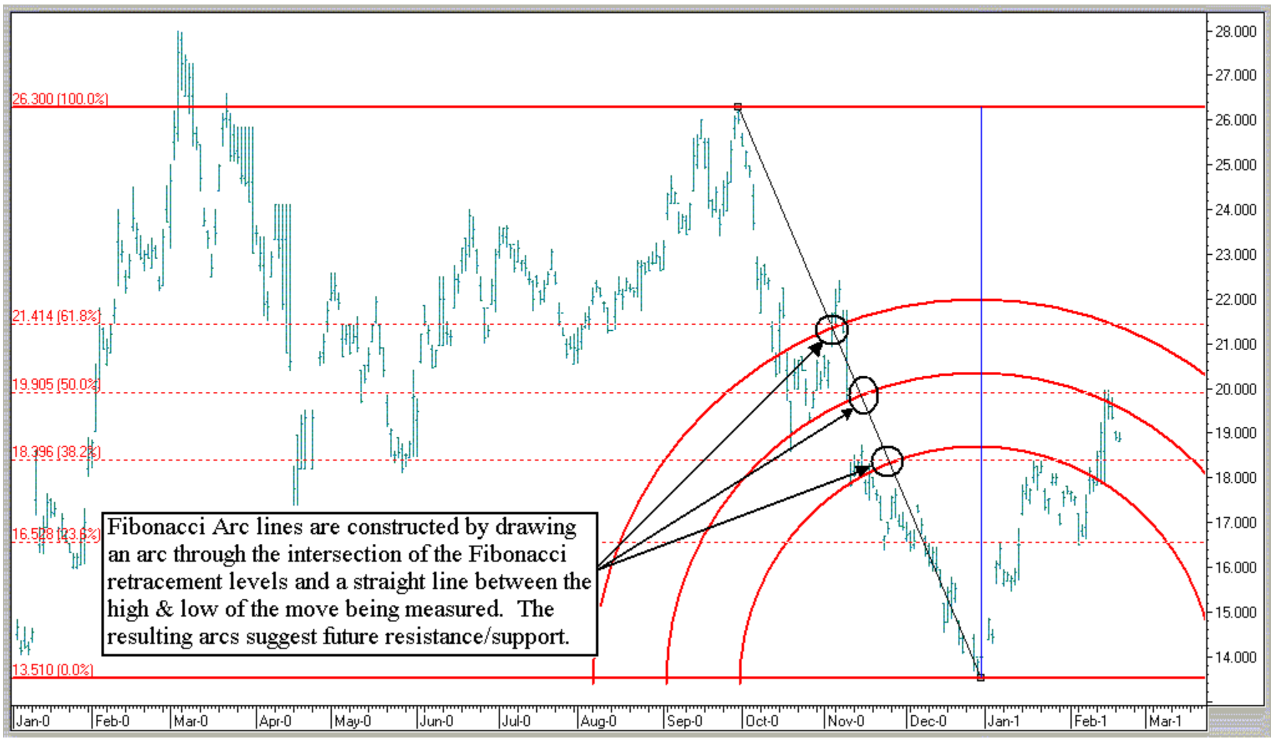
The above examples and discussion relating to Elliott Wave Theory represent only a minor scratch in the surface of this broad topic as our main interest here is the connection between Fibonacci and market movements.

A slight twist on the Fibonacci retracement concept is provided by Fibonacci Fan lines. As the chart below shows, these fan lines use Fibonacci retracement levels to provide a dynamic level of support or resistance based on the direction and magnitude of the move that is being retraced.



Once a support fan line has been broken it will often be seen providing resistance with the market spending time chopping between here and the support provided by the next support fan line, and vice versa.

Another method similar to the Fibonacci Fan is the Fibonacci Arc. As shown below this involves drawing an arc through the point where the Fibonacci retracement levels intersect a straight line drawn between the high & the low. The resulting arc suggests areas of future resistance/support.



Another application of both the individual Fibonacci numbers and the relationships between the numbers (The Golden Ratio) is found in the arena of time measurement and cycle analysis. Significant turning points can often be anticipated by counting forward from previous important highs or lows by a specific (Fibonacci) number of days, weeks or months. As well as significant tops and bottoms occasionally being a specific number of (Fibonacci) days, weeks or months apart, there is often a connection also in the ratio of time taken for the formation of one wave and the time taken for the formation of the following wave or an alternate wave. For example, a sharp pullback might develop in 38.2% of the time taken for the market to complete the prior rally. An in depth analysis of this area is something that would be more closely associated with a study of the works of W. D. Gann and his techniques of squaring price and time.

EPILOGUE

No doubt Leonardo Pisano would be amazed to see just how far and wide his influence has spread in the past 750-760 years. It is quite likely that he would also be surprised to discover that it was a relatively obscure problem that was posed in his book *Liber Abaci* that lead to a French mathematician, Edouard Lucas, naming the Fibonacci sequence in Leonardo's honor more than 600 years after his death.

While there is little empirical evidence to support the statistical reliability of the regular formation of many of the relationships mentioned above, observations over time have shown that these relationships do appear often enough to be considered a useful part of the technical analysts' toolbox. This may seem like a weak connection for people who require a clear cause & effect relationship but as with most areas of market analysis, we are left to anticipate future movements based on the balance of probabilities derived from prior observations of the market.

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